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TWO APPROACHES TO CLOSING THE EQUATIONS OF MOMENTUM AND HEAT TRANSFER IN TURBULENT CHEMICALLY REACTIVE GAS-DISPERSED FLOWS ON THE STABILIZED PORTION OF AN AXISYMMETRIC CHANNEL

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Consideration is given to two systems of equations of transfer of correlations of particle-velocity and temperature pulsations, which differ in the method of closing. A closed description of the heat transfer of the solid phase is carried out at the level of equations for second moments in the first case and for third moments in the second case. A comparative analysis of the two methods of calculation of the ascending nonisothermal flow of a gas suspension is made based on numerical investigations.

Representation of the field of a nonisothermal turbulent dispersed flow in the form of averaged and pulsatory motions observed in actual flows has given rise to the second moments of particle-velocity and temperature pulsations in the initial momentum and energy equations. The fundamental difficulty in the path of development of this trend lies with modeling unknown correlation terms of the above equations. We must note that theoretical models of this class of flows have been developed to a lesser extent than the methods of calculation of momentum transfer in the solid phase in isothermal flows. Turbulent heat transfer in the dispersed phase is modeled, as a rule, on the basis of the gradient representations $\langle t'_p v'_p \rangle = \frac{\lambda_p dt_p}{c_p \rho_p \partial r}$ [1] or algebraic locally equilibrium models in which the turbulent heat flux in the solid phase is directly related to the Reynolds pulsation heat transfer in the carrier medium $\langle t'_g v'_g \rangle$ [2]. The wish to overcome the boundedness of the above models gave rise to more complex turbulence models based on additional differential equations of transfer of the moments of particle-velocity and temperature pulsations. In [3], the variable $\langle t'_p v'_p \rangle$ was computed from the equation of transfer of the variable itself. Triple correlations of the type $\langle t'_p v'^2_p \rangle$ and $\langle t'_p w'^2_p \rangle$ that were present in this equation were found using gradient models. In [4], a chain of truncated equations of transfer of the third moments of particle-velocity and temperature pulsations was constructed with the kinetic equation for the probability-density function; the algebraic relations relating the triple correlations to those double were determined from these equations. This made it possible to obtain a closed description of the heat transfer of the dispersed phase at the level of equations for second moments.

In the present work, we make an attempt (probably for the first time) to obtain, within the framework of the Eulerian approach, i.e., in the case of the so-called two-fluid models, a closed description of the heat transfer of the solid phase at the level of equations for triple correlations. For this purpose, using the computational procedure developed [5, 6], we have obtained a chain of axisymmetric averaged equations of transfer of the second, third, and fourth moments of particle-velocity and temperature pulsations on the portion of stabilized ascending motion of a gas suspension with allowance for convective and radiant heat exchange and for the drag force. The equations for the correlations of fourth order are closed on the basis of representation of the fifth moments present in these equations as the sum of the products of those second and third. This enabled us to obtain, from the equations for fourth moments, algebraic relations expressing the fourth correlations by the second and third moments and their gradients. Furthermore, in this work, we compare two methods of closing: the first method closes at the level of equations for second moments, whereas the second method, at the level of equations of transfer of triple correlations.

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In constructing a system of equations of transfer of the averaged and pulsation characteristics of a nonisothermal chemically reactive two-phase flow, we take the following simplifying prerequisites: (1) the process is stationary; (2) the stoichiometric scheme of the reactions includes one heterogeneous reaction $C + O_2 = CO_2$; (3) the volume concentration of the solid phase and the mass concentrations of carbon in particles and oxygen in the carrier medium are constant over the entire cross section of the tube; (4) the dispersed phase consists of monodisperse spherical coke-ash particles; (5) there is no averaged radial and transversal phase motion on the portion of stabilized flow of the gas suspension and the averaged parameters do not change in the axial direction.

Basic Equations. With allowance for what has been said above, we can represent the system of equations of momentum and energy transfer in a two-phase medium in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\rho_g r\,(\eta_{t,g}+\eta_g)\,\frac{\partial u_g}{\partial r}\right]-\frac{\partial P}{\partial z}-F_{az}=0\,,\quad \frac{\rho_p\beta}{r}\frac{\partial}{\partial r}\left(r\eta_{t,p}\frac{\partial u_p}{\partial r}\right)+F_{az}-\rho_p\beta g=0\,, \tag{1}$$

$$-\frac{\rho_p\beta c_p}{r}\frac{\partial\,(r\,\langle t'_p v'_p\rangle)}{\partial r}-\alpha_\Sigma\,(t_p-t_g)\,\frac{6\beta}{\delta}+Q=0\,,\quad \alpha_\Sigma=\alpha_{con}+\alpha_{rad}\,, \tag{2}$$

$$\frac{c_g}{r}\frac{\partial}{\partial r}\left[\rho_g r\left(\frac{\eta_{t,g}}{\mathrm{Pr}_{t,g}}+\frac{\eta_g}{\mathrm{Pr}_g}\right)\frac{\partial t_g}{\partial r}\right]+10^{-3}\left[u_g\frac{\partial P}{\partial z}+F_{az}\,(u_g-u_p)+\rho_g\eta_{t,g}\left(\frac{\partial u_g}{\partial r}\right)^2\right]$$
$$+\,\alpha_\Sigma\,(t_p-t_g)\,\frac{6\beta}{\delta}=0\,,\quad Q=\frac{6S\gamma C_{O_2}\beta C_C q}{(S+\gamma)\,\delta}\,,\quad \gamma=L\exp\left[-\frac{E}{(t_p+273)\,H}\right], \tag{3}$$

$$\frac{\rho_g}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\eta_{t,g}}{\sigma_k}+\eta_g\right)\frac{\partial k_g}{\partial r}\right]+\rho_g\eta_{t,g}\left(\frac{\partial u_g}{\partial r}\right)^2-\rho_g\,(\varepsilon_g+\varepsilon_p)+G=0\,. \tag{4}$$

The left-hand sides of Eqs. (1) include the viscous and Reynolds stresses, the pressure gradient, and the drag and gravity forces. Terms allowing for pulsation heat transfer in the solid phase, heat exchange between the carrier medium and the dispersed phase, and heat release due to the heterogeneous chemical reaction appear in (2). The molecular and turbulent transfer of the gas flow, the work of Reynolds stresses and pressure and interphase-interaction forces, and radiant and convective heat exchange between the gas and particles are allowed for in (3). The first term of Eq. (4) describes the diffusion of the pulsation energy of the carrier medium, the second term describes its generation due to the averaged-motion energy, the third and fourth terms describe its dissipation due to the viscosity of the gas and the presence of the solid phase in it, and the last term describes the generation of turbulent energy in the trails behind particles. The unknown correlation $\langle t'_p v'_p\rangle$, which in turn is dependent on the second and third moments, is present in Eq. (2). Therefore, we must construct the transfer equations for the correlations sought to close the above system of equations.

Equations of Transfer of the Second Moments. To derive the equations of transfer of the variables $\langle t'_p v'_p\rangle$ and $\langle t'_p w'_p\rangle$ we must primarily obtain the equations of pulsatory motion and energy of particles. The equations of pulsatory motion of the dispersed phase along the radial and transversal axes have been obtained in [7]. With allowance made for the axial symmetry of the problem ($\partial/\partial\varphi = 0$), we can write the mentioned equations in the form

$$u_p\frac{\partial v'_p}{\partial z}+u'_p\frac{\partial v_p}{\partial z}+u'_p\frac{\partial v'_p}{\partial z}+v_p\frac{\partial v'_p}{\partial r}+v'_p\frac{\partial v_p}{\partial r}+v'_p\frac{\partial v'_p}{\partial r}-\frac{1}{r}\,w'_p w'_p-\frac{\partial\,\langle u'_p v'_p\rangle}{\partial z}$$
$$-\,\frac{\partial\,(r\,\langle v'_p v'_p\rangle)}{r\partial r}+\frac{1}{r}\,\langle w'_p w'_p\rangle=\frac{F'_{ar}}{\rho_p\beta}\,, \tag{5}$$

$$u_p \frac{\partial w'_p}{\partial z} + u'_p \frac{\partial w'_p}{\partial z} + v_p \frac{\partial w'_p}{\partial r} + v'_p \frac{\partial w'_p}{\partial r} + \frac{1}{r}(v_p w'_p + w'_p v'_p) - \frac{\partial \langle u'_p w'_p \rangle}{\partial z}$$

$$- \frac{\partial (r \langle v'_p w'_p \rangle)}{r \partial r} - \frac{1}{r} \langle w'_p v'_p \rangle = \frac{F'_{a\varphi}}{\rho_p \beta}, \qquad (6)$$

where

$$F'_{ar} = \frac{\rho_p \beta}{\tau}(v'_g - v'_p); \quad F'_{a\varphi} = \frac{\rho_p \beta}{\tau}(w'_g - w'_p). \qquad (7)$$

Applying the Reynolds procedure to the actual equation of particle energy

$$\hat{u}_p \frac{\partial \hat{t}_p}{\partial z} + \hat{u}_p \frac{\partial \hat{t}_p}{\partial r} = \frac{6\alpha_\Sigma}{\rho_p c_p \delta}(\hat{t}_g - \hat{t}_p) + Q \quad (\hat{\alpha}_\Sigma = \alpha_\Sigma, \ \hat{c}_p = c_p, \ \partial/\partial\varphi = 0, \ \hat{\delta} = \delta, \ \hat{Q} = Q) \qquad (8)$$

we arrive at the pulsation equation of heat transfer in the solid phase:

$$u_p \frac{\partial t'_p}{\partial z} + u'_p \frac{\partial t'_p}{\partial z} + u'_p \frac{\partial t_p}{\partial z} + v_p \frac{\partial t'_p}{\partial r} + v'_p \frac{\partial t'_p}{\partial r} + v'_p \frac{\partial t_p}{\partial r} - \frac{\partial \langle t'_p u'_p \rangle}{\partial z} - \frac{\partial (r \langle t'_p v'_p \rangle)}{r \partial r} = \frac{6\alpha_\Sigma}{\rho_p c_p \delta}(t'_g - t'_p). \qquad (9)$$

To construct the equation of transfer of the correlation moment $\langle t'_p v'_p \rangle$ we must multiply Eq. (5) by the quantity t'_p and Eq. (9) by v'_p and thereafter combine the resulting equations:

$$u_p \frac{\partial t'_p v'_p}{\partial z} + v_p \frac{\partial t'_p v'_p}{\partial r} + u'_p \frac{\partial t'_p v'_p}{\partial z} + v'_p \frac{\partial t'_p v'_p}{\partial r} + t'_p u'_p \frac{\partial v_p}{\partial z} + u'_p v'_p \frac{\partial t_p}{\partial z} + t'_p v'_p \frac{\partial v_p}{\partial r} + v'^2_p \frac{\partial t_p}{\partial r} - \frac{w'^2_p t'_p}{r}$$

$$= \frac{F'_{ar} t'_p}{\rho_p \beta} + \frac{6\alpha_\Sigma}{\rho_p c_p \delta}(t'_g v'_p - t'_p v'_p). \qquad (10)$$

Using expressions (7) and the pulsation continuity equation premultiplied by the quantity $t'_p v'_p$, we transform (10), after which we carry out averaging in the equation obtained. On the portion of stabilized motion of the two-phase flow, the equation of transfer of the second moment $\langle t'_p v'_p \rangle$ has the form

$$\frac{\partial (r \langle t'_p v'^2_p \rangle)}{r \partial r} + \frac{\langle v'^2_p \rangle \partial t_p}{\partial r} - \frac{\langle t'_p w'^2_p \rangle}{r} = \frac{1}{\tau}(\langle t'_p v'_g \rangle - \langle t'_p v'_p \rangle) + \frac{6\alpha_\Sigma}{\rho_p c_p \delta}(\langle t'_g v'_p \rangle - \langle t'_p v'_p \rangle). \qquad (11)$$

In the same manner, we can obtain the equation of transfer of the correlation $\langle t'_p w'_p \rangle$:

$$\frac{\partial (r \langle t'_p w'_p v'_p \rangle)}{r \partial r} + \frac{\langle w'_p v'_p \rangle \partial t_p}{\partial r} + \frac{\langle t'_p w'_p v'_p \rangle}{r} = \frac{1}{\tau}(\langle t'_p w'_g \rangle - \langle t'_p w'_p \rangle) + \frac{6\alpha_\Sigma}{\rho_p c_p \delta}(\langle t'_g w'_p \rangle - \langle t'_p w'_p \rangle). \qquad (12)$$

The mixed moments (gas–particle) present in Eqs. (11) and (12) are determined in terms of the correlations of the carrier flow in a locally homogeneous approximation in accordance with the recommendations of [2], whereas the second moments of pulsations of the translational velocity of the dispersed phase $\langle v'^2_p \rangle$ and $\langle w'_p v'_p \rangle$ are computed according to [5].

Closing at the Level of the Equations of Transfer of the Correlations $\langle t'_p v'_p \rangle$ and $\langle t'_p w'_p \rangle$. The unknown third moments $\langle t'_p w'_p v'_p \rangle$, $\langle t'_p v'^2_p \rangle$, and $\langle t'_p w'^2_p \rangle$ appear in Eqs. (11) and (12). To compute them we construct the equations of transfer of the correlations sought. Let us use the equation for the third moment $\langle t'_p w'_p v'_p \rangle$ to illustrate the derivation of these equations. We multiply the pulsation equation (5) by the quantity w'_{p} and Eq. (6) by v'_{p} and combine the resulting equations:

$$u_{\text{p}}\frac{\partial w'_{\text{p}}v'_{\text{p}}}{\partial z}+v_{\text{p}}\frac{\partial w'_{\text{p}}v'_{\text{p}}}{\partial r}+u'_{\text{p}}\frac{\partial w'_{\text{p}}v'_{\text{p}}}{\partial z}+v'_{\text{p}}\frac{\partial w'_{\text{p}}v'_{\text{p}}}{\partial r}+u'_{\text{p}}w'_{\text{p}}\frac{\partial v_{\text{p}}}{\partial z}+w'_{\text{p}}v'_{\text{p}}\frac{\partial v_{\text{p}}}{\partial r}-\frac{w'^3_{\text{p}}}{r}+\frac{w'_{\text{p}}\langle w'^2_{\text{p}}\rangle}{r}$$

$$+\frac{v_{\text{p}}w'_{\text{p}}v'_{\text{p}}}{r}+\frac{v'^2_{\text{p}}w'_{\text{p}}}{r}-\frac{v'_{\text{p}}\langle w'_{\text{p}}v'_{\text{p}}\rangle}{r}-\frac{w'_{\text{p}}\partial\langle u'_{\text{p}}v'_{\text{p}}\rangle}{\partial z}-\frac{w'_{\text{p}}\partial(r\langle v'^2_{\text{p}}\rangle)}{r\partial r}-\frac{v'_{\text{p}}\partial\langle u'_{\text{p}}w'_{\text{p}}\rangle}{\partial z}$$

$$-\frac{v'_{\text{p}}\partial(r\langle w'_{\text{p}}v'_{\text{p}}\rangle)}{r\partial r}=\frac{F'_{\text{a}r}w'_{\text{p}}+F'_{\text{a}\varphi}v'_{\text{p}}}{\rho_{\text{p}}\beta}. \tag{13}$$

Next, we multiply (13) by the quantity t'_{p} and Eq. (9) by $w'_{\text{p}}v'_{\text{p}}$, after which we sum up these equations. Disregarding mixed triple correlations (gas–particle) and using expressions (7), we can reduce the equation of transfer of the quantity $\langle t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}\rangle$ to the following form:

$$u_{\text{p}}\frac{\partial t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}}{\partial z}+v_{\text{p}}\frac{\partial t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}}{\partial r}+u'_{\text{p}}\frac{\partial t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}}{\partial z}+v'_{\text{p}}\frac{\partial t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}}{\partial r}+t'_{\text{p}}u'_{\text{p}}w'_{\text{p}}\frac{\partial v_{\text{p}}}{\partial z}+t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}\frac{\partial v_{\text{p}}}{\partial r}$$

$$+u'_{\text{p}}w'_{\text{p}}v'_{\text{p}}\frac{\partial t_{\text{p}}}{\partial z}+w'_{\text{p}}v'^2_{\text{p}}\frac{\partial t_{\text{p}}}{\partial r}-\frac{t'_{\text{p}}w'^3_{\text{p}}}{r}+\frac{t'_{\text{p}}w'_{\text{p}}\langle w'^2_{\text{p}}\rangle}{r}+\frac{v_{\text{p}}t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}}{r}+\frac{t'_{\text{p}}w'_{\text{p}}v'^2_{\text{p}}}{r}-\frac{v'_{\text{p}}t'_{\text{p}}\langle w'_{\text{p}}v'_{\text{p}}\rangle}{r}$$

$$-\frac{t'_{\text{p}}w'_{\text{p}}\partial\langle u'_{\text{p}}v'_{\text{p}}\rangle}{\partial z}-\frac{t'_{\text{p}}w'_{\text{p}}\partial(r\langle v'^2_{\text{p}}\rangle)}{r\partial r}-\frac{t'_{\text{p}}v'_{\text{p}}\partial\langle u'_{\text{p}}w'_{\text{p}}\rangle}{\partial z}-\frac{t'_{\text{p}}v'_{\text{p}}\partial(r\langle w'_{\text{p}}v'_{\text{p}}\rangle)}{r\partial r}-\frac{w'_{\text{p}}v'_{\text{p}}\partial\langle t'_{\text{p}}u'_{\text{p}}\rangle}{\partial z}$$

$$-\frac{w'_{\text{p}}v'_{\text{p}}\partial(r\langle t'_{\text{p}}v'_{\text{p}}\rangle)}{r\partial r}=-\psi_2t'_{\text{p}}w'_{\text{p}}v'_{\text{p}},\quad \psi_2=\frac{6\alpha_\Sigma}{\rho_{\text{p}}c_{\text{p}}\delta}+\frac{2}{\tau}. \tag{14}$$

We transform (14) using the pulsation continuity equation premultiplied by the quantity $t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}$. Then we carry out averaging in the final equation. On the portion of stabilized motion of the gas suspension, we write the equation of transfer of the sought correlation $\langle t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}\rangle$:

$$\frac{\partial(r\langle t'_{\text{p}}w'_{\text{p}}v'^2_{\text{p}}\rangle)}{r\partial r}+\frac{\langle v'^2_{\text{p}}w'_{\text{p}}\rangle\,\partial t_{\text{p}}}{\partial r}-\frac{\langle t'_{\text{p}}w'^3_{\text{p}}\rangle}{r}+\frac{\langle t'_{\text{p}}w'_{\text{p}}\rangle\,\langle w'^2_{\text{p}}\rangle}{r}+\frac{\langle t'_{\text{p}}w'_{\text{p}}v'^2_{\text{p}}\rangle}{r}$$

$$-\frac{\langle t'_{\text{p}}v'_{\text{p}}\rangle\,\langle w'_{\text{p}}v'_{\text{p}}\rangle}{r}-\frac{\langle t'_{\text{p}}w'_{\text{p}}\rangle\,\partial(r\langle v'^2_{\text{p}}\rangle)}{r\partial r}-\frac{\langle t'_{\text{p}}v'_{\text{p}}\rangle\,\partial(r\langle w'_{\text{p}}v'_{\text{p}}\rangle)}{r\partial r}-\frac{\langle w'_{\text{p}}v'_{\text{p}}\rangle\,\partial(r\langle t'_{\text{p}}v'_{\text{p}}\rangle)}{r\partial r}=-\psi_2\langle t'_{\text{p}}w'_{\text{p}}v'_{\text{p}}\rangle. \tag{15}$$

Analogously we can obtain transfer equations for the correlations $\langle t'_{\text{p}}v'^2_{\text{p}}\rangle$ and $\langle t'_{\text{p}}w'^2_{\text{p}}\rangle$. We give these equations without derivation:

$$\frac{\partial(r\langle t'_{\text{p}}v'^3_{\text{p}}\rangle)}{2r\partial r}+\frac{\langle v'^3_{\text{p}}\rangle\,\partial t_{\text{p}}}{2\partial r}-\frac{\langle t'_{\text{p}}w'^2_{\text{p}}v'_{\text{p}}\rangle}{r}-\frac{\langle v'^2_{\text{p}}\rangle\,\partial(r\langle t'_{\text{p}}v'_{\text{p}}\rangle)}{2r\partial r}-\frac{\langle t'_{\text{p}}v'_{\text{p}}\rangle\,\partial(r\langle v'^2_{\text{p}}\rangle)}{r\partial r}$$

$$+\frac{\langle t'_{\text{p}}v'_{\text{p}}\rangle\,\langle w'^2_{\text{p}}\rangle}{r}=-\psi_1\langle t'_{\text{p}}v'^2_{\text{p}}\rangle,\quad \psi_1=\frac{3\alpha_\Sigma}{\rho_{\text{p}}c_{\text{p}}\delta}+\frac{1}{\tau}, \tag{16}$$

$$\frac{\partial\left(r\,\langle t'_{\rm p}w'^2_{\rm p}v'_{\rm p}\rangle\right)}{2r\partial r}+\frac{\langle t'_{\rm p}w'^2_{\rm p}v'_{\rm p}\rangle}{r}-\frac{\langle t'_{\rm p}w'_{\rm p}\rangle\,\langle w'_{\rm p}v'_{\rm p}\rangle}{r}-\frac{\langle t'_{\rm p}w'_{\rm p}\rangle\,\partial\left(r\,\langle w'_{\rm p}v'_{\rm p}\rangle\right)}{r\partial r}$$

$$+\,\langle w'^2_{\rm p}v'_{\rm p}\rangle\,\frac{\partial t_{\rm p}}{2\partial r}-\frac{\langle w'^2_{\rm p}\rangle\,\partial\left(r\,\langle t'_{\rm p}v'_{\rm p}\rangle\right)}{2r\partial r}=-\,\psi_1\,\langle t'_{\rm p}w'^2_{\rm p}\rangle\,. \tag{17}$$

In (15)–(16), there are the fourth moments which can be expressed similarly to [5]:

$$\langle t'_{\rm p}v'^3_{\rm p}\rangle=3\,\langle v'^2_{\rm p}\rangle\,\langle t'_{\rm p}v'_{\rm p}\rangle\,,\quad\langle t'_{\rm p}w'^2_{\rm p}v'_{\rm p}\rangle=2\,\langle w'_{\rm p}v'_{\rm p}\rangle\,\langle t'_{\rm p}w'_{\rm p}\rangle+\langle w'^2_{\rm p}\rangle\,\langle t'_{\rm p}v'_{\rm p}\rangle\,,$$

$$\langle t'_{\rm p}w'_{\rm p}v'^2_{\rm p}\rangle=2\,\langle w'_{\rm p}v'_{\rm p}\rangle\,\langle t'_{\rm p}v'_{\rm p}\rangle+\langle v'^2_{\rm p}\rangle\,\langle t'_{\rm p}w'_{\rm p}\rangle\,,\quad\langle t'_{\rm p}w'^3_{\rm p}\rangle=3\,\langle w'^2_{\rm p}\rangle\,\langle t'_{\rm p}w'_{\rm p}\rangle\,. \tag{18}$$

Substituting (18) into (15)–(17), after simple transformations we will have

$$\langle t'_{\rm p}w'_{\rm p}v'_{\rm p}\rangle=-\frac{1}{\psi_2}\left[\frac{\langle v'^2_{\rm p}\rangle\,\partial\,\langle t'_{\rm p}w'_{\rm p}\rangle}{\partial r}+\frac{\langle t'_{\rm p}v'_{\rm p}\rangle\,\partial\,\langle w'_{\rm p}v'_{\rm p}\rangle}{\partial r}+\frac{\langle w'_{\rm p}v'_{\rm p}\rangle\,\langle t'_{\rm p}v'_{\rm p}\rangle}{r}\right.$$

$$\left.+\frac{\langle w'_{\rm p}v'_{\rm p}\rangle\,\partial\,\langle t'_{\rm p}v'_{\rm p}\rangle}{\partial r}+\frac{\langle v'^2_{\rm p}w'_{\rm p}\rangle\,\partial t_{\rm p}}{\partial r}-\frac{2\,\langle w'^2_{\rm p}\rangle\,\langle t'_{\rm p}w'_{\rm p}\rangle}{r}+\frac{\langle v'^2_{\rm p}\rangle\,\langle t'_{\rm p}w'_{\rm p}\rangle}{r}\right], \tag{19}$$

$$\langle t'_{\rm p}v'^2_{\rm p}\rangle=-\frac{1}{\psi_1}\left[\frac{\langle v'^2_{\rm p}\rangle\,\partial\,\langle t'_{\rm p}v'_{\rm p}\rangle}{\partial r}+\frac{\langle t'_{\rm p}v'_{\rm p}\rangle\,\partial\,\langle v'^2_{\rm p}\rangle}{2\partial r}+\frac{\langle v'^3_{\rm p}\rangle\,\partial t_{\rm p}}{2\partial r}-\frac{2\,\langle w'_{\rm p}v'_{\rm p}\rangle\,\langle w'_{\rm p}t'_{\rm p}\rangle}{r}\right], \tag{20}$$

$$\langle t'_{\rm p}w'^2_{\rm p}\rangle=-\frac{1}{\psi_1}\left[\frac{\langle w'_{\rm p}v'_{\rm p}\rangle\,\partial\,\langle t'_{\rm p}w'_{\rm p}\rangle}{\partial r}+\frac{\langle t'_{\rm p}v'_{\rm p}\rangle\,\partial\,\langle w'^2_{\rm p}\rangle}{2\partial r}+\frac{\langle w'_{\rm p}v'_{\rm p}\rangle\,\langle t'_{\rm p}w'_{\rm p}\rangle}{r}\right.$$

$$\left.+\frac{\langle w'^2_{\rm p}v'_{\rm p}\rangle\,\partial t_{\rm p}}{2\partial r}+\frac{\langle w'^2_{\rm p}\rangle\,\langle t'_{\rm p}v'_{\rm p}\rangle}{r}\right]. \tag{21}$$

To obtain a system of parabolic equations of transfer of the second moments we must substitute expressions (19)–(21) into (11) and (12). Finally,

the equation of transfer of the quantity $\langle t'_{\rm p}v'_{\rm p}\rangle$ is

$$-\frac{\partial}{r\partial r}\left(\frac{r\,\langle v'^2_{\rm p}\rangle}{\psi_1}\,\frac{\partial\,\langle t'_{\rm p}v'_{\rm p}\rangle}{\partial r}\right)-\frac{\partial}{2r\partial r}\left(\frac{r\,\langle t'_{\rm p}v'_{\rm p}\rangle}{\psi_1}\,\frac{\partial\,\langle v'^2_{\rm p}\rangle}{\partial r}\right)-\frac{\partial}{2r\partial r}\left(\frac{r\,\langle v'^3_{\rm p}\rangle}{\psi_1}\,\frac{\partial t_{\rm p}}{\partial r}\right)$$

$$+\frac{2\partial}{r\partial r}\left(\frac{\langle w'_{\rm p}v'_{\rm p}\rangle\,\langle t'_{\rm p}w'_{\rm p}\rangle}{\psi_1}\right)+\langle v'^2_{\rm p}\rangle\,\frac{\partial t_{\rm p}}{\partial r}+\frac{\langle w'_{\rm p}v'_{\rm p}\rangle\,\partial\,\langle t'_{\rm p}w'_{\rm p}\rangle}{\psi_1 r\partial r}+\frac{\langle t'_{\rm p}v'_{\rm p}\rangle\,\partial\,\langle w'^2_{\rm p}\rangle}{2\psi_1 r\partial r}$$

$$+\frac{\langle w'_{\rm p}v'_{\rm p}\rangle\,\langle t'_{\rm p}w'_{\rm p}\rangle}{\psi_1 r^2}+\frac{\langle v'_{\rm p}w'^2_{\rm p}\rangle}{2\psi_1}\,\frac{\partial t_{\rm p}}{r\partial r}+\frac{\langle w'^2_{\rm p}\rangle\,\langle t'_{\rm p}v'_{\rm p}\rangle}{\psi_1 r^2}=\frac{1}{\tau}\left(\langle t'_{\rm p}v'_{\rm g}\rangle-\langle t'_{\rm p}v'_{\rm p}\rangle\right)+\frac{6\alpha_\Sigma}{\rho_{\rm p}c_{\rm p}\delta}\left(\langle t'_{\rm g}v'_{\rm p}\rangle-\langle t'_{\rm p}v'_{\rm p}\rangle\right) \tag{22}$$

and the equation of transfer of the quantity $\langle t'_{\rm p}w'_{\rm p}\rangle$ is

$$-\frac{\partial}{r\partial r}\left(\frac{r\,\langle v_p'^2\rangle}{\psi_2}\frac{\partial\,\langle t_p'w_p'\rangle}{\partial r}\right)-\frac{\partial}{r\partial r}\left(\frac{r\,\langle t_p'v_p'\rangle}{\psi_2}\frac{\partial\,\langle w_p'v_p'\rangle}{\partial r}\right)-\frac{\partial}{r\partial r}\left(\frac{\langle t_p'v_p'\rangle\,\langle w_p'v_p'\rangle}{\psi_2}\right)$$

$$-\frac{\partial}{r\partial r}\left(\frac{r\,\langle w_p'v_p'\rangle}{\psi_2}\frac{\partial\,\langle t_p'v_p'\rangle}{\partial r}\right)-\frac{\partial}{r\partial r}\left(\frac{r\,\langle v_p'^2w_p'\rangle}{\psi_2}\frac{\partial t_p}{\partial r}\right)+\frac{2\partial}{r\partial r}\left(\frac{\langle t_p'w_p'\rangle\,\langle w_p'^2\rangle}{\psi_2}\right)$$

$$-\frac{\partial}{r\partial r}\left(\frac{\langle v_p'^2\rangle\,\langle t_p'w_p'\rangle}{\psi_2}\right)+\langle w_p'v_p'\rangle\frac{\partial t_p}{\partial r}-\frac{\langle v_p'^2\rangle}{\psi_2}\frac{\partial\,\langle t_p'w_p'\rangle}{r\partial r}-\frac{\langle t_p'v_p'\rangle}{\psi_2}\frac{\partial\,\langle w_p'v_p'\rangle}{r\partial r}$$

$$-\frac{\langle t_p'v_p'\rangle\,\langle w_p'v_p'\rangle}{\psi_2 r^2}-\frac{\langle w_p'v_p'\rangle}{\psi_2}\frac{\partial\,\langle t_p'v_p'\rangle}{r\partial r}-\frac{\langle v_p'^2w_p'\rangle}{\psi_2}\frac{\partial t_p}{r\partial r}+\frac{2\,\langle w_p'^2\rangle\,\langle t_p'w_p'\rangle}{\psi_2 r^2}$$

$$-\frac{\langle v_p'^2\rangle\,\langle t_p'w_p'\rangle}{\psi_2 r^2}=\frac{\langle t_p'w_g'\rangle-\langle t_p'w_p'\rangle}{\tau}+\frac{6\alpha_\Sigma\,(\langle t_g'w_p'\rangle-\langle t_p'w_p'\rangle)}{\rho_p c_p\delta}\,. \tag{23}$$

The unknown moments of second $\langle w_p'w_p'\rangle$ and third $\langle v_p'v_p'v_p'\rangle$, $\langle v_p'w_p'w_p'\rangle$, $\langle v_p'v_p'w_p'\rangle$ orders of particle-velocity pulsations, which are determined according to [5], appear in Eqs. (22) and (23). Thus, we have obtained a closed description of the heat transfer of the solid phase at the level of equations for second correlations.

Closing at the Level of the Equations of Transfer of the Triple Correlations $\langle t_p'w_p'v_p'\rangle$, $\langle t_p'v_p'^2\rangle$, and $\langle t_p'w_p'^2\rangle$. The fourth moments $\langle t_p'w_p'^2v_p'\rangle$, $\langle t_p'v_p'^3\rangle$, $\langle t_p'w_p'v_p'^2\rangle$, and $\langle t_p'w_p'^3\rangle$ for which, as has been mentioned above, we must obtain their own transfer equations appear in Eqs. (15)–(17). Let us use the equation for the variable $\langle t_p'w_p'v_p'^2\rangle$ to illustrate derivation of these equations. We multiply the pulsation equation (14) by the quantity v_p' and Eq. (5) by $t_p'w_p'v_p'$, after which we sum up these equations. We transform the resulting equation using expression (7) and the pulsation continuity equation premultiplied by the quantity $t_p'w_p'v_p'^2$. Then we carry out averaging in the final equation. Disregarding mixed correlation moments (gas–particle), we write the equation of transfer of the sought quantity $\langle t_p'w_p'v_p'^2\rangle$ for the portion of steady-state motion of the gas-dispersed flow:

$$\frac{\partial\,(r\,\langle t_p'w_p'v_p'^3\rangle)}{2r\partial r}+\frac{\langle t_p'w_p'v_p'^3\rangle}{2r}-\frac{\langle w_p'v_p'\rangle\,\langle t_p'v_p'^2\rangle}{2r}-\frac{\langle t_p'v_p'w_p'^3\rangle}{r}+\frac{\langle w_p'v_p'^3\rangle\,\partial t_p}{2\partial r}$$

$$-\frac{\langle t_p'v_p'^2\rangle\,\partial\,(r\,\langle w_p'v_p'\rangle)}{2r\partial r}-\frac{\langle w_p'v_p'^2\rangle\,\partial\,(r\,\langle t_p'v_p'\rangle)}{2r\partial r}-\frac{\langle t_p'w_p'v_p'\rangle\,\partial\,(r\,\langle v_p'^2\rangle)}{r\partial r}$$

$$+\frac{\langle w_p'^2\rangle\,\langle t_p'w_p'v_p'\rangle}{r}=-\psi_4\,\langle t_p'w_p'v_p'^2\rangle\,,\quad \psi_4=3\left(\frac{\alpha_\Sigma}{\rho_p c_p\delta}+\frac{1}{2\tau}\right). \tag{24}$$

Equation (24) contains the fifth moments which, similarly to [6], can be represented in the form of the sum of the products of correlations of second and third moments. With allowance for this fact, we transform the equation indicated to the form

$$\langle t_p'w_p'v_p'^2\rangle=-\frac{1}{\psi_4}\left[\frac{\langle v_p'^2\rangle\,\partial\,\langle t_p'w_p'v_p'\rangle}{\partial r}+\frac{\langle w_p'v_p'\rangle\,\partial\,\langle t_p'v_p'^2\rangle}{2\partial r}+\frac{\langle t_p'v_p'\rangle\,\partial\,\langle w_p'v_p'^2\rangle}{2\partial r}\right.$$

$$+\frac{\langle v_p'^2\rangle\,\langle t_p'w_p'v_p'\rangle}{r}+\frac{\langle t_p'v_p'\rangle\,\langle w_p'v_p'^2\rangle}{2r}-\frac{3\,\langle w_p'v_p'\rangle\,\langle t_p'w_p'^2\rangle}{r}-\frac{\langle t_p'v_p'\rangle\,\langle w_p'^3\rangle}{r}$$

$$+\frac{\langle w_p'^2\rangle\,\langle t_p'w_p'v_p'\rangle}{r}+\frac{3\,\langle v_p'^2\rangle\,\langle w_p'v_p'\rangle\,\partial t_p}{2\partial r}\Bigg]. \tag{25}$$

In a similar manner, we can obtain transfer equations for the remaining correlations sought:

$$\langle t_p'v_p'^3\rangle=-\frac{1}{\psi_3}\Bigg[\frac{\langle v_p'^2\rangle\,\partial\,\langle t_p'v_p'^2\rangle}{\partial r}+\frac{\langle t_p'v_p'\rangle\,\partial\,\langle v_p'^3\rangle}{3\partial r}+\frac{\langle v_p'^2\rangle^2\,\partial t_p}{\partial r}-\frac{\langle v_p'^2\rangle\,\langle t_p'w_p'^2\rangle}{r}$$

$$-\frac{2\,\langle w_p'v_p'\rangle\,\langle t_p'w_p'v_p'\rangle}{r}-\frac{\langle w_p'^2v_p'\rangle\,\langle t_p'v_p'\rangle}{r}+\frac{\langle t_p'v_p'^2\rangle\,\langle w_p'^2\rangle}{r}\Bigg],\quad \psi_3=\frac{2\alpha_\Sigma}{\rho_p c_p\delta}+\frac{1}{\tau}, \tag{26}$$

$$\langle t_p'w_p'^2v_p'\rangle=-\frac{1}{\psi_4}\Bigg[\frac{\langle v_p'^2\rangle\,\partial\,\langle t_p'w_p'^2\rangle}{2\partial r}+\frac{\langle w_p'v_p'\rangle\,\partial\,\langle t_p'w_p'v_p'\rangle}{\partial r}+\frac{\langle t_p'v_p'\rangle\,\partial\,\langle v_p'w_p'^2\rangle}{2\partial r}$$

$$+\frac{\langle v_p'^2\rangle\,\langle t_p'w_p'^2\rangle}{r}+\frac{\langle w_p'v_p'\rangle\,\langle t_p'w_p'v_p'\rangle}{r}+\frac{\langle w_p'^2v_p'\rangle\,\langle t_p'v_p'\rangle}{r}-\frac{\langle w_p'^2\rangle\,\langle t_p'w_p'^2\rangle}{r}$$

$$-\frac{\langle w_p'^3\rangle\,\langle t_p'w_p'\rangle}{2r}+\frac{\langle v_p'^2\rangle\,\langle w_p'^2\rangle\,\partial t_p}{2\partial r}+\frac{\langle w_p'v_p'\rangle^2\,\partial t_p}{\partial r}\Bigg], \tag{27}$$

$$\langle t_p'w_p'^3\rangle=-\frac{1}{\psi_3}\Bigg[\frac{\langle w_p'v_p'\rangle\,\partial\,\langle t_p'w_p'^2\rangle}{\partial r}+\frac{\langle t_p'v_p'\rangle\,\partial\,\langle w_p'^3\rangle}{3\partial r}+\frac{2\,\langle w_p'v_p'\rangle\,\langle t_p'w_p'^2\rangle}{r}$$

$$+\frac{\langle w_p'^3\rangle\,\langle t_p'v_p'\rangle}{r}+\frac{\langle w_p'v_p'\rangle\,\langle w_p'^2\rangle\,\partial t_p}{\partial r}\Bigg]. \tag{28}$$

Substituting (25)–(28) into Eqs. (15)–(17), we obtain a system of parabolic equations of transfer of the third moments:

the equation of transfer of the quantity $\langle t_p'w_p'^2\rangle$

$$-\frac{\partial}{4r\partial r}\left(\frac{r\,\langle v_p'^2\rangle}{\psi_4}\frac{\partial\,\langle t_p'w_p'^2\rangle}{\partial r}\right)-\frac{\partial}{2r\partial r}\left(\frac{r\,\langle w_p'v_p'\rangle}{\psi_4}\frac{\partial\,\langle t_p'w_p'v_p'\rangle}{\partial r}\right)$$

$$-\frac{\partial}{4r\partial r}\left(\frac{r\,\langle t_p'v_p'\rangle}{\psi_4}\frac{\partial\,\langle w_p'^2v_p'\rangle}{\partial r}\right)-\frac{\partial}{2r\partial r}\left(\frac{\langle v_p'^2\rangle\,\langle t_p'w_p'^2\rangle}{\psi_4}\right)-\frac{\partial}{2r\partial r}\left(\frac{\langle w_p'v_p'\rangle\,\langle t_p'w_p'v_p'\rangle}{\psi_4}\right)$$

$$-\frac{\partial}{2r\partial r}\left(\frac{\langle t_p'v_p'\rangle\,\langle w_p'^2v_p'\rangle}{\psi_4}\right)+\frac{\partial}{r\partial r}\left(\frac{\langle w_p'^2\rangle\,\langle t_p'w_p'^2\rangle}{\psi_4}\right)+\frac{\partial}{4r\partial r}\left(\frac{\langle w_p'^3\rangle\,\langle t_p'w_p'\rangle}{\psi_4}\right)$$

$$-\frac{\partial}{4r\partial r}\left(\frac{r\,\langle v_p'^2\rangle\,\langle w_p'^2\rangle}{\psi_4}\frac{\partial t_p}{\partial r}\right)-\frac{\partial}{2r\partial r}\left(\frac{r\,\langle w_p'v_p'\rangle^2}{\psi_4}\frac{\partial t_p}{\partial r}\right)-\frac{\langle v_p'^2\rangle}{2\psi_4}\frac{\partial\,\langle t_p'w_p'^2\rangle}{r\partial r}$$

$$-\frac{\langle w'_p v'_p\rangle}{\psi_4}\frac{\partial\langle t'_p w'_p v'_p\rangle}{r\partial r}-\frac{\langle t'_p v'_p\rangle}{2\psi_4}\frac{\partial\langle w'^2_p v'_p\rangle}{r\partial r}-\frac{\langle v'^2_p\rangle\langle t'_p w'^2_p\rangle}{\psi_4 r^2}-\frac{\langle w'_p v'_p\rangle\langle t'_p w'_p v'_p\rangle}{\psi_4 r^2}$$

$$-\frac{\langle w'^2_p v'_p\rangle\langle t'_p v'_p\rangle}{\psi_4 r^2}+\frac{\langle w'^2_p\rangle\langle t'_p w'^2_p\rangle}{\psi_4 r^2}+\frac{\langle w'^3_p\rangle\langle t'_p w'_p\rangle}{2\psi_4 r^2}-\frac{\langle v'^2_p\rangle\langle w'^2_p\rangle\partial t_p}{2\psi_4 r\partial r}-\frac{\langle w'_p v'_p\rangle^2}{\psi_4}\frac{\partial t_p}{r\partial r}$$

$$-\frac{\langle t'_p w'_p\rangle\langle w'_p v'_p\rangle}{r}-\frac{\langle t'_p w'_p\rangle\partial(r\langle w'_p v'_p\rangle)}{r\partial r}+\frac{\langle w'^2_p v'_p\rangle}{2}\frac{\partial t_p}{\partial r}-\frac{\langle w'^2_p\rangle}{2}\frac{\partial(r\langle t'_p v'_p\rangle)}{r\partial r}=-\psi_1\langle t'_p w'^2_p\rangle\,; \qquad (29)$$

the equation of transfer of the quantity $\langle t'_p v'^2_p\rangle$

$$-\frac{\partial}{2r\partial r}\left(\frac{r\langle v'^2_p\rangle}{\psi_3}\frac{\partial\langle t'_p v'^2_p\rangle}{\partial r}\right)-\frac{\partial}{6r\partial r}\left(\frac{r\langle t'_p v'_p\rangle}{\psi_3}\frac{\partial\langle v'^3_p\rangle}{\partial r}\right)-\frac{\partial}{2r\partial r}\left(\frac{r\langle v'^2_p\rangle^2}{\psi_3}\frac{\partial t_p}{\partial r}\right)$$

$$+\frac{\partial}{2r\partial r}\left(\frac{\langle v'^2_p\rangle\langle t'_p w'^2_p\rangle}{\psi_3}\right)+\frac{\partial}{r\partial r}\left(\frac{\langle w'_p v'_p\rangle\langle t'_p w'_p v'_p\rangle}{\psi_3}\right)+\frac{\partial}{2r\partial r}\left(\frac{\langle w'^2_p v'_p\rangle\langle t'_p v'_p\rangle}{\psi_3}\right)$$

$$-\frac{\partial}{2r\partial r}\left(\frac{\langle t'_p v'^2_p\rangle\langle w'^2_p\rangle}{\psi_3}\right)+\langle v'^3_p\rangle\frac{\partial t_p}{2\partial r}+\frac{\langle v'^2_p\rangle}{2\psi_4}\frac{\partial\langle t'_p w'^2_p\rangle}{r\partial r}+\frac{\langle w'_p v'_p\rangle}{\psi_4}\frac{\partial\langle t'_p w'_p v'_p\rangle}{r\partial r}$$

$$+\frac{\langle t'_p v'_p\rangle\partial\langle w'^2_p v'_p\rangle}{2\psi_4 r\partial r}+\frac{\langle v'^2_p\rangle\langle t'_p w'^2_p\rangle}{\psi_4 r^2}+\frac{\langle w'_p v'_p\rangle\langle t'_p w'_p v'_p\rangle}{\psi_4 r^2}+\frac{\langle w'^2_p v'_p\rangle\langle t'_p v'_p\rangle}{\psi_4 r^2}$$

$$-\frac{\langle w'^2_p\rangle\langle t'_p w'^2_p\rangle}{\psi_4 r^2}-\frac{\langle w'^3_p\rangle\langle t'_p w'_p\rangle}{2\psi_4 r^2}+\frac{\langle v'^2_p\rangle\langle w'^2_p\rangle}{2\psi_4}\frac{\partial t_p}{r\partial r}+\frac{\langle w'_p v'_p\rangle^2}{\psi_4}\frac{\partial t_p}{r\partial r}$$

$$-\frac{\langle v'^2_p\rangle}{2}\frac{\partial(r\langle t'_p v'_p\rangle)}{r\partial r}-\langle t'_p v'_p\rangle\frac{\partial(r\langle v'^2_p\rangle)}{r\partial r}+\frac{\langle t'_p v'_p\rangle\langle w'^2_p\rangle}{r}=-\psi_1\langle t'_p v'^2_p\rangle\,, \qquad (30)$$

and the equation of transfer of the quantity $\langle t'_p w'_p v'_p\rangle$

$$-\frac{\partial}{r\partial r}\left(\frac{r\langle v'^2_p\rangle}{\psi_4}\frac{\partial\langle t'_p w'_p v'_p\rangle}{\partial r}\right)-\frac{\partial}{2r\partial r}\left(\frac{r\langle w'_p v'_p\rangle}{\psi_4}\frac{\partial\langle t'_p v'^2_p\rangle}{\partial r}\right)$$

$$-\frac{\partial}{2r\partial r}\left(\frac{r\langle t'_p v'_p\rangle}{\psi_4}\frac{\partial\langle v'^2_p w'_p\rangle}{\partial r}\right)-\frac{\partial}{r\partial r}\left(\frac{\langle v'^2_p\rangle\langle t'_p w'_p v'_p\rangle}{\psi_4}\right)-\frac{\partial}{2r\partial r}\left(\frac{\langle t'_p v'_p\rangle\langle v'^2_p w'_p\rangle}{\psi_4}\right)$$

$$+\frac{3\partial}{r\partial r}\left(\frac{\langle w'_p v'_p\rangle\langle t'_p w'^2_p\rangle}{\psi_4}\right)+\frac{\partial}{r\partial r}\left(\frac{\langle t'_p v'_p\rangle\langle w'^3_p\rangle}{\psi_4}\right)-\frac{\partial}{r\partial r}\left(\frac{\langle w'^2_p\rangle\langle t'_p w'_p v'_p\rangle}{\psi_4}\right)$$

$$-\frac{3\partial}{2r\partial r}\left(\frac{r\langle v'^2_p\rangle\langle w'_p v'_p\rangle}{\psi_4}\frac{\partial t_p}{\partial r}\right)+\langle v'^2_p w'_p\rangle\frac{\partial t_p}{\partial r}+\frac{\langle w'_p v'_p\rangle}{\psi_3}\frac{\partial\langle t'_p w'^2_p\rangle}{r\partial r}$$

$$+\frac{\langle t'_p v'_p\rangle}{3\psi_3}\frac{\partial\langle w'^3_p\rangle}{r\partial r}+\frac{2\langle w'_p v'_p\rangle\langle t'_p w'^2_p\rangle}{\psi_3 r^2}+\frac{\langle t'_p v'_p\rangle\langle w'^3_p\rangle}{\psi_3 r^2}+\frac{\langle w'_p v'_p\rangle\langle w'^2_p\rangle}{\psi_3}\frac{\partial t_p}{r\partial r}$$

$$+\frac{\langle t'_p w'_p\rangle\langle w'^2_p\rangle}{r}-\frac{\langle v'^2_p\rangle}{\psi_4}\frac{\partial\langle t'_p w'_p v'_p\rangle}{r\partial r}-\frac{\langle w'_p v'_p\rangle}{2\psi_4}\frac{\partial\langle t'_p v'^2_p\rangle}{r\partial r}-\frac{\langle t'_p v'_p\rangle}{2\psi_4}\frac{\partial\langle v'^2_p w'_p\rangle}{r\partial r}$$

$$-\frac{\langle v'^2_p\rangle\langle t'_p w'_p v'_p\rangle}{\psi_4 r^2}-\frac{\langle t'_p v'_p\rangle\langle v'^2_p w'_p\rangle}{2\psi_4 r^2}+\frac{3\langle w'_p v'_p\rangle\langle t'_p w'^2_p\rangle}{\psi_4 r^2}+\frac{\langle t'_p v'_p\rangle\langle w'^3_p\rangle}{\psi_4 r^2}$$

$$-\frac{\langle w'^2_p\rangle\langle t'_p w'_p v'_p\rangle}{\psi_4 r^2}-\frac{3\langle v'^2_p\rangle\langle w'_p v'_p\rangle}{2\psi_4}\frac{\partial t_p}{r\partial r}-\frac{\langle t'_p v'_p\rangle\langle w'_p v'_p\rangle}{r}-\langle t'_p w'_p\rangle\frac{\partial(r\langle v'^2_p\rangle)}{r\partial r}$$

$$-\frac{\langle t'_p v'_p\rangle\,\partial(r\langle w'_p v'_p\rangle)}{r\partial r}-\frac{\langle w'_p v'_p\rangle\,\partial(r\langle t'_p v'_p\rangle)}{r\partial r}=-\psi_2\langle t'_p w'_p v'_p\rangle\,. \quad (31)$$

The unknown second and third moments of pulsations of the translational particle velocity which appear in Eqs. (29)–(31) can be determined according to [5, 7]. Thus, we have obtained a closed description of the heat transfer of the solid phase at the level of equations for triple correlations.

We have obtained two closed systems of equations, (1)–(4) and (19)–(23) and (1)–(4), (11), (12), and (25)–(31), which describe the behavior of the ascending, nonisothermal, chemically reactive two-phase flow and differ in the method of closing. Boundary conditions on the channel axis (r = 0) are specified for these systems for reasons of symmetry

$$r=0:\quad \partial u_g/\partial r=\partial k_g/\partial r=\partial t_g/\partial r=\partial u_p/\partial r=\partial\langle t'_p v'_p\rangle/\partial r=\partial\langle t'_p w'_p\rangle/\partial r=0\,,$$

$$\partial\langle t'_p w'^2_p\rangle/\partial r=\partial\langle t'_p v'^2_p\rangle/\partial r=\partial\langle t'_p w'_p v'_p\rangle/\partial r=0\,,$$

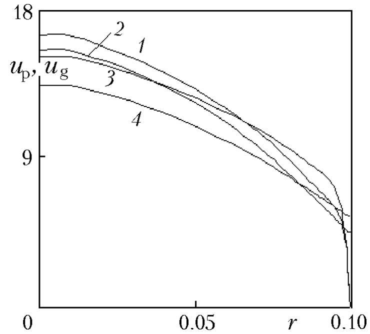
and those on the tube wall (r = R) are prescribed by the relations

$$r=R:\quad u_g=k_g=0\,,\quad t_g=t_w\,,\quad u_p=\frac{\delta(7K_n-2K_\tau-5)\,\partial u_p}{24\sqrt{2}\,\beta(1-K_\tau)\,\partial r}\,, \quad (32)$$

$$\partial\langle t'_p w'^2_p\rangle/\partial r=\partial\langle t'_p v'^2_p\rangle/\partial r=\partial\langle t'_p w'_p v'_p\rangle/\partial r=\partial\langle t'_p v'_p\rangle/\partial r=\partial\langle t'_p w'_p\rangle/\partial r=0\,.$$

The above systems of equations with boundary conditions (32) were numerically integrated by the marching method with iterations on a nonuniform grid clustering at the channel wall; the pressure gradient was eliminated using the well-known method [8]. Based on the algorithms described, we developed programs with which numerical investigations of the aerodynamics, heat and mass exchange, and combustion of monodisperse coke-ash particles of anthracite culm on the portion of stabilized ascending motion of the gas suspension were carried out.

Certain Calculation Results and Their Discussion. We consider results of calculations of three variants for the following initial data: β = 0.0012, ρ_p = 1600 kg/m^3, $u_{g,m}$ = 10.5 m/sec, Z_{O_2} = 0.23, t_w = 650^{o}C, K_τ = 0.3, K_n = 0.5, and R = 0.1 m. Variants I and II: $\delta = 0.2\cdot10^{-3}$ m and C_C = 0.114; III: $\delta = 0.3\cdot10^{-3}$ m and C_C = 0.183. The first variant was calculated with the system of equations (1)–(4), (11), (12), and (25)–(31), whereas the second and third variants were calculated with (1)–(4) and (19)–(23). The calculated material is illustrated in Figs. 1–6, where the profiles of the averaged and pulsation characteristics of the nonisothermal gas-dispersed flow are presented. Figure 1 shows the distribution of the averaged velocities of the gas and the dispersed phase over the flow cross section. In the axial zone where the drag force is directed upward ($F_{az} \sim (u_g - u_p) > 0$), the dispersed phase lags behind the gas the



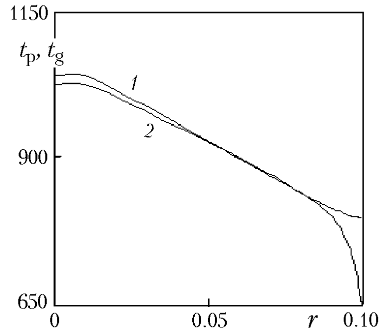
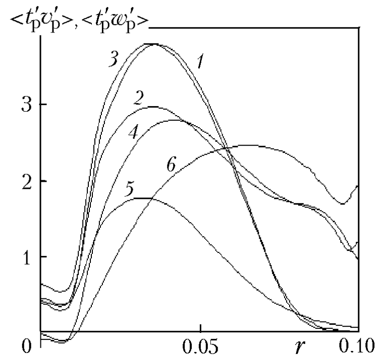


Fig. 1. Profiles of longitudinal velocities of the gas and particles: variant I) 1) u_g and 2) u_p; variant III) 3) u_g and 4) u_p.

Fig. 2. Profiles of averaged temperatures of the gas and coke-ash particles for variant III: 1) t_p and 2) t_g.



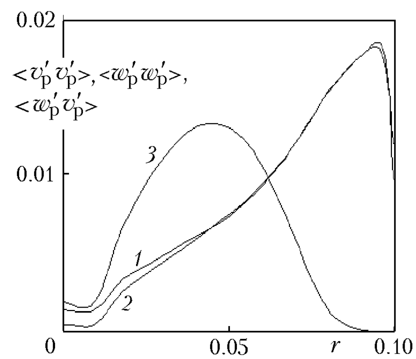


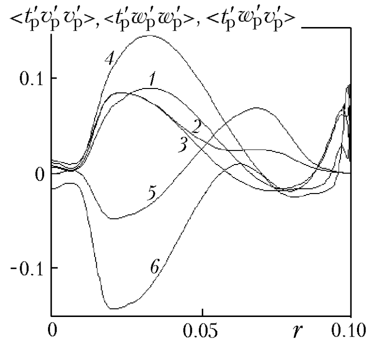
Fig. 3. Distribution of the second moments of dispersed-phase-velocity and temperature pulsations: variant I) 1) $\langle t'_p w'_p \rangle$ and 2) $\langle t'_p v'_p \rangle$; II) 3) $\langle t'_p w'_p \rangle$ and 4) $\langle t'_p v'_p \rangle$; III) 5) $\langle t'_p w'_p \rangle$ and 6) $\langle t'_p v'_p \rangle$.

Fig. 4. Profiles of the second moments of dispersed-phase-velocity pulsations: variant I) 1) $\langle w'_p w'_p \rangle$ and 2) $\langle v'_p v'_p \rangle$; II) 3) $\langle w'_p v'_p \rangle$.

more, the larger the particle. In the wall region, the interphase-slip velocity $u_g - u_p$ is negative and the bridging of particles is related primarily to their random motion (first term of the second equation of (1)). From Fig. 1, it is clear that the velocity profile of the carrier medium becomes more filled with increase in the particle diameter (curves 1 and 3 are compared). This is due to the generation of the turbulent energy of the gaseous phase because of the separation of the flow behind the large particle in flow (fifth terms of Eq. (4)).

Figure 2 gives the calculated values of the averaged gas and particle temperatures on the portion of stabilized two-phase flow. Change in the temperature of coke-ash particles over the tube cross section is mainly dependent on two factors: heat release due to the chemical reaction $C + O_2 = CO_2$ and heat exchange between the carrier medium and the dispersed phase (second and third terms of Eq. (2)). The first factor turns out to be prevailing in the axial zone, which ensures the efficient burnout of a solid fuel, whose intensity grows toward the flow axis. The second factor is determining near the wall (due to the sharp increase in the temperature head $t_p - t_g$), which finally leads to a cooling of the coke-ash particles. From Fig. 2, it follows that the character of change in the $t_p(r)$ curves is close to the character of change in the dependences $t_g(r)$. In the central part of the tube, the temperatures of both phases are close, whereas in the wall region, the function $t_p(r)$ decreases much more slowly than the $t_g(r)$ curve under the influence of pulsation transfer in the solid phase (first term of Eq. (2)).

Figure 3 shows the profiles of the mixed moments of the pulsation characteristics of particles $\langle t'_p w'_p \rangle$ and $\langle t'_p v'_p \rangle$ over the flow cross section. The balance of the terms of Eq. (23) shows that the dominant role in formation of



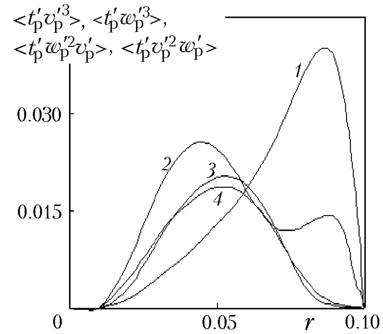


Fig. 5. Distribution of the third moments of particle-velocity and temperature pulsations on the portion of stabilized flow of the gas suspension: variant I) 1) $\langle t'_p v'^2_p \rangle$, 2) $\langle t'_p w'_p v'_p \rangle$, and 3) $\langle t'_p w'^2_p \rangle$; variant II) 4) $\langle t'_p v'^2_p \rangle$, 5) $\langle t'_p w'_p v'_p \rangle$, and 6) $\langle t'_p w'^2_p \rangle$.

Fig. 6. Distribution of the fourth moments of particle-velocity and temperature pulsations over the flow cross section for variant I: 1) $\langle t'_p v'^3_p \rangle$, 2) $\langle t'_p w'^2_p v'_p \rangle$, 3) $\langle t'_p w'^3_p \rangle$, and 4) $\langle t'_p w'_p v'^2_p \rangle$.

the profile $\langle t'_p w'_p \rangle(r)$ (curve 3) is played by the eighth $\langle w'_p v'_p \rangle \dfrac{dt_p}{dr}$, sixteenth $\dfrac{\langle t'_p w'_g \rangle}{\tau}$, and eighteenth $\dfrac{6\alpha_\Sigma \langle t'_g w'_p \rangle}{\rho_p c_p \delta}$ terms of the above equation. Rapid growth in the $\langle t'_p w'_p \rangle(r)$ curve in the range 0.0095 m $< r <$ 0.033 m is related to the increase in the functions $\langle w'_p v'_p \rangle(r)$ (Fig. 4, curve 3), $\langle t'_p w'_g \rangle(r)$, and $\langle t'_g w'_p \rangle(r)$ and to the decrease in the dependence $t_p(r)$ in this zone. The decrease in the dependence $\langle t'_p w'_p \rangle(r)$ on the descending portion 0.033 m $< r <$ 0.085 m is due to the decrease in the tangential Reynolds stress $\langle w'_p v'_p \rangle$ and the mixed correlations $\langle t'_p w'_g \rangle$ and $\langle t'_g v'_p \rangle$ in the interval in question.

Figure 5 gives results of calculations of the third moments of particle-velocity and temperature pulsations, obtained with expressions (19)–(21) and the transfer equations (29)–(31). From Fig. 5, it is clear that the character of change in the dependence $\langle t'_p v'^2_p \rangle(r)$, calculated with the algebraic expression (20) (curve 4), is close to the character of change in the same dependence found from (30) (curve 1). Unlike the character of the distribution of the functions $\langle t'_p v'^2_p \rangle(r)$, no similarity is observed between the $\langle t'_p w'_p v'_p \rangle(r)$ curves computed with the approximate formula (19) and the parabolic equation (31) (curves 2 and 5 are compared). As far as the behavior of the functions $\langle t'_p w'^2_p \rangle(r)$ is concerned (curves 3 and 6 are compared), curves 3 and 6 are sharply different in the axial zone $0 < r <$ 0.066 m, whereas in the peripheral region 0.066 m $< r$, these dependences are similar.

Figure 6 gives the calculated values of the mixed correlations of fourth order of particle-velocity and temperature pulsations on the portion of steady-state ascending motion of the gas suspension. In Fig. 6, it is seen that the function $\langle t'_p v'^3_p \rangle(r)$ has a pronounced maximum at the point r = 0.088 m (curve 1); the presence of this maximum is due to the influence of the third term $\langle v'^2_p \rangle^2 \partial t_p/(\psi_3 \partial r)$ of Eq. (26). On the ascending branch $0.0095 < r < 0.088$ m, the character of the $\langle t'_p v'^3_p \rangle(r)$ curve is determined by the increase in the function $\langle v'_p v'_p \rangle(r)$ (Fig. 4, curve 2) and the decrease in the $t_p(r)$ curve. On the descending portion $r >$ 0.088 m, the decrease in the dependence $\langle t'_p v'^3_p \rangle(r)$ is related to the decrease in the correlation $\langle v'_p v'_p \rangle$ and in the absolute value of the derivative $|\partial t_p/\partial r|$ in this zone. From a comparison of the dependences $\langle t'_p v'^2_p \rangle(r)$ (Fig. 5, curve 1) and $\langle t'_p v'^3_p \rangle(r)$ (Fig. 6, curve 1), it is clear that in the peripheral part of the channel $0 < r <$ 0.066 m, the values of the variable $\langle t'_p v'^2_p \rangle$ are higher than those of the moment $\langle t'_p v'^3_p \rangle$, whereas in the peripheral region $0.066 < r < 0.093$ m, the correlation $\langle t'_p v'^3_p \rangle$ exceeds the quantity $\langle t'_p v'^2_p \rangle$; this suggests that it is necessary to allow for the fourth moments of particle-velocity and temperature pulsations in calculations of nonisothermal two-phase turbulent flows.

Thus, the presented mathematical model of aerodynamics and physicochemical processes reflects the basic regularities of the processes of mass, momentum, and energy transfer in reactive two-phase systems on the portion of stabilized flow of a gas suspension. The computational procedure proposed makes it possible to obtain detailed information on the distribution of the most important parameters of the working process, which can be useful in designing flow-type chemical reactors.

NOTATION

C, concentration; *c*, heat capacity, kJ/(kg·K); *E*, activation energy, kJ/kmole; *F*, force, $kg/(sec^2{\cdot}m^2)$; *G*, generation of the turbulent gas energy in the trails behind particles, $kg/(sec^3{\cdot}m)$; *g*, free-fall acceleration, m/sec^2; *H*, universal gas constant, kJ/(kmole·K); *K*, velocity recovery factor (restitution factor); *k*, kinetic pulsation energy, m^2/sec^2; *L*, preexponential factor, m/sec; *P*, gas pressure, N/m^2; Pr, Prandtl number; *Q*, heat release due to the heterogeneous chemical reaction, $kJ/(sec{\cdot}m^3)$; *q*, thermal effect of the reaction $C + O_2 = CO_2$, kJ/kmole; *R*, channel radius, m; *r*, *z*, and φ, radial, longitudinal, and transversal coordinates, m; *S*, mass-exchange coefficient, m/sec; *t*, temperature, ^oC; *u*, *v*, and *w*, averaged components of the velocity vector, m/sec; *Z*, mass fraction of the component of the gas mixture; α, coefficient of mass exchange between the gas and the particle, $kJ/(sec{\cdot}m^2{\cdot}K)$; β, true volume concentration of particles; γ, reaction-rate constant, m/sec; δ, particle diameter, m; ε, pulsation-energy dissipation, m^2/sec^3; η, kinematic viscosity, m^2/sec; λ, thermal conductivity, kJ/(sec·m·K); ρ, density, kg/m^3; σ, empirical constant; τ, dynamic-relaxation time, sec; ψ_1, ψ_2, ψ_3, and ψ_4, coefficients, sec^{-1}. Subscripts: a, aerodynamic resistance (drag) of a particle; con, convective heat exchange; g, gas; m, mean (over the cross section); n, normal; p, particle; rad, radiant heat exchange; t, pulsations; w, channel wall; τ, tangential; Σ, total heat exchange. Superscripts: $'$, pulsation component in time averaging; $\langle\ \rangle$, time averaging; ^, actual values.

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